MATHEMATICS (MATH)

MATH 120. Elementary Functions and Analytic Geometry. 3 Units.
Polynomial, rational, exponential, logarithmic, and trigonometric functions (emphasis on computation, graphing, and location of roots) straight lines and conic sections. Primarily a precalculus course for the student without a good background in trigonometric functions and graphing and/or analytic geometry. Prereq: Three years of high school mathematics. Prereq: No previous credit for MATH 121 or MATH 125.

MATH 121. Calculus for Science and Engineering I. 4 Units.
Functions, analytic geometry of lines and polynomials, limits, derivatives of algebraic and trigonometric functions. Definite integral, antiderivatives, fundamental theorem of calculus, change of variables. Recommended preparation: Three and one half years of high school mathematics. Credit for at most one of MATH 121, MATH 123 and MATH 125 can be applied to hours required for graduation. Counts as a CAS Quantitative Reasoning course. Counts as a Quantitative Reasoning course. Prereq: MATH 120 or a score of 30 on the mathematics diagnostic test or exempt from the mathematics diagnostic test.

MATH 122. Calculus for Science and Engineering II. 4 Units.
Continuation of MATH 121. Exponentials and logarithms, growth and decay, inverse trigonometric functions, related rates, basic techniques of integration, area and volume, polar coordinates, parametric equations. Taylor polynomials and Taylor's theorem. Credit for at most one of MATH 122, MATH 124, and MATH 126 can be applied to hours required for graduation. Prereq: MATH 121, MATH 123 or MATH 126.

MATH 124. Calculus II. 4 Units.
Review of differentiation. Techniques of integration, and applications of the definite integral. Parametric equations and polar coordinates. Taylor's theorem. Sequences, series, power series. Complex arithmetic. Introduction to multivariable calculus. Credit for at most one of MATH 122, MATH 124, and MATH 126 can be applied to hours required for graduation. Prereq: MATH 121 and placement by department.

MATH 125. Math and Calculus Applications for Life, Managerial, and Social Sci I. 4 Units.
Discrete and continuous probability; differential and integral calculus of one variable; graphing, related rates, maxima and minima. Integration techniques, numerical methods, volumes, areas. Applications to the physical, life, and social sciences. Students planning to take more than two semesters of introductory mathematics should take MATH 121. Recommended preparation: Three and one half years of high school mathematics. Credit for at most one of MATH 121, MATH 123, and MATH 125 can be applied to hours required for graduation. Counts as a CAS Quantitative Reasoning course. Counts as a Quantitative Reasoning course. Prereq: MATH 120 or a score of 30 on the Mathematics diagnostic test or exempt from the Mathematics diagnostic test.

MATH 126. Math and Calculus Applications for Life, Managerial, and Social Sci II. 4 Units.
Continuation of MATH 125 covering differential equations, multivariable calculus, discrete methods. Partial derivatives, maxima and minima for functions of two variables, linear regression. Differential equations; first and second order equations, systems, Taylor series methods; Newton's method; difference equations. Credit for at most one of MATH 122, MATH 124, and MATH 126 can be applied to hours required for graduation. Prereq: MATH 121, MATH 123 or MATH 125.

MATH 201. Introduction to Linear Algebra for Applications. 3 Units.
Matrix operations, systems of linear equations, vector spaces, subspaces, bases and linear independence, eigenvalues and eigenvectors, diagonalization of matrices, linear transformations, determinants. Less theoretical than MATH 307. Appropriate for majors in science, engineering, economics. Prereq: MATH 122, MATH 124 or MATH 126.

MATH 223. Calculus for Science and Engineering III. 3 Units.
Introduction to vector algebra; lines and planes. Functions of several variables: partial derivatives, gradients, chain rule, directional derivative, maxima/minima. Multiple integrals, cylindrical and spherical coordinates. Derivatives of vector valued functions, velocity and acceleration. Vector fields, line integrals, Green's theorem. Credit for at most one of MATH 223 and MATH 227 can be applied to hours required for graduation. Prereq: MATH 122 or MATH 124.

MATH 224. Elementary Differential Equations. 3 Units.
A first course in ordinary differential equations. First order equations and applications, linear equations with constant coefficients, linear systems, Laplace transforms, numerical methods of solution. Credit for at most one of MATH 224 and MATH 228 can be applied to hours required for graduation. Prereq: MATH 223 or MATH 227.

MATH 227. Calculus III. 3 Units.
Vector algebra and geometry. Linear maps and matrices. Calculus of vector valued functions. Derivatives of functions of several variables. Multiple integrals. Vector fields and line integrals. Credit for at most one of MATH 223 and MATH 227 can be applied to hours required for graduation. Prereq: MATH 124 and placement by the department.

MATH 228. Differential Equations. 3 Units.
Elementary ordinary differential equations: first order equations; linear systems; applications; numerical methods of solution. Credit for at most one of MATH 224 and MATH 228 can be applied to hours required for graduation. Prereq: MATH 227 or placement by the department.

MATH 301. Undergraduate Reading Course. 1 - 3 Units.
Students must obtain the approval of a supervising professor before registration. More than one credit hour must be approved by the undergraduate committee of the department.

MATH 302. Departmental Seminar. 3 Units.
A seminar devoted to understanding the formulation and solution of mathematical problems. SAGES Department Seminar. Students will investigate, from different possible viewpoints, via case studies, how mathematics advances as a discipline--what mathematicians do. The course will largely be in a seminar format. There will be two assignments involving writing in the style of the discipline. Enrollment by permission (limited to majors depending on demand). Counts as a SAGES Departmental Seminar course.

MATH 303. Elementary Number Theory. 3 Units.
Primes and divisibility, theory of congruencies, and number theoretic functions. Diophantine equations, quadratic residue theory, and other topics determined by student interest. Emphasis on problem solving (formulating conjectures and justifying them). Prereq: MATH 122 or MATH 124.

MATH 304. Discrete Mathematics. 3 Units.
A general introduction to basic mathematical terminology and the techniques of abstract mathematics in the context of discrete mathematics. Topics introduced are mathematical reasoning, Boolean connectives, deduction, mathematical induction, sets, functions and relations, algorithms, graphs, combinatorial reasoning. Offered as CSDS 302, ECSE 302 and MATH 304. Prereq: MATH 122 or MATH 124 or MATH 126.

MATH 305. Introduction to Advanced Mathematics. 3 Units.
A course on the theory and practice of writing, and reading mathematics. Main topics are logic and the language of mathematics, proof techniques, set theory, and functions. Additional topics may include introductions to number theory, group theory, topology, or other areas of advanced mathematics. Prereq: MATH 122, MATH 124 or MATH 126.

MATH 307. Linear Algebra. 3 Units.
A course in linear algebra that studies the fundamentals of vector spaces, inner product spaces, and linear transformations on an axiomatic basis. Topics include: solutions of linear systems, matrix algebra over the real and complex numbers, linear independence, bases and dimension, eigenvalues and eigenvectors, singular value decomposition, and determinants. Other topics may include least squares, general inner product and normed spaces, orthogonal projections, finite dimensional spectral theorem. This course is required of all students majoring in mathematics and applied mathematics. More theoretical than MATH 201. Prereq: MATH 122 or MATH 124.

MATH 308. Introduction to Abstract Algebra. 3 Units.
A first course in abstract algebra, studied on an axiomatic basis. The major algebraic structures studied are groups, rings and fields. Topics include homomorphisms and quotient structures. This course is required of all students majoring in mathematics. It is helpful, but not necessary, for a student to have taken MATH 307 before MATH 308. Prereq: MATH 122 or MATH 124.

MATH 309. Sets, Logic, and Categories. 3 Units.
An introduction to three different perspectives on foundations of mathematics. Topics include naive set theory, axiomatic set theory, propositional logic, formal proofs, categories and universality, elementary theory of the category of sets. Counts as a Disciplinary Communication course. Prereq: MATH 224 or MATH 228.

MATH 319. Applied Probability and Stochastic Processes for Biology. 3 Units.
Applications of probability and stochastic processes to biological systems. Mathematical topics will include: introduction to discrete and continuous probability spaces (including numerical generation of pseudo random samples from specified probability distributions), Markov processes in discrete and continuous time with discrete and continuous sample spaces, point processes including homogeneous and inhomogeneous Poisson processes and Markov chains on graphs, and diffusion processes including Brownian motion and the Ornstein-Uhlenbeck process. Biological topics will be determined by the interests of the students and the instructor. Likely topics include: stochastic ion channels, molecular motors and stochastic ratchets, actin and tubulin polymerization, random walk models for neural spike trains, bacterial chemotaxis, signaling and genetic regulatory networks, and stochastic predator-prey dynamics. The emphasis will be on practical simulation and analysis of stochastic phenomena in biological systems. Numerical methods will be developed using a combination of MATLAB, the R statistical package, MCell, and/or URDME, at the discretion of the instructor. Student projects will comprise a major part of the course. Offered as BIOL 319, ECSE 319, MATH 319, SYBB 319, BIOL 419, EBME 419, MATH 419, PHOL 419, and SYBB 419. Prereq: MATH 224 or MATH 223 and BIOL 300 or BIOL 306 and MATH 201 or MATH 307 or consent of instructor.

MATH 321. Fundamentals of Analysis I. 3 Units.
Abstract mathematical reasoning in the context of analysis in Euclidean space. Introduction to formal reasoning, sets and functions, and the number systems. Sequences and series; Cauchy sequences and convergence. Required for all mathematics majors. Additional work required for graduate students. (May not be taken for graduate credit by graduate students in the Department of Mathematics.) Offered as MATH 321 and MATH 421. Prereq: MATH 223 or MATH 227.

MATH 322. Fundamentals of Analysis II. 3 Units.
Continuation of MATH 321. Point-set topology in metric spaces with attention to n-dimensional space; completeness, compactness, connectedness, and continuity of functions. Topics in sequences, series of functions, uniform convergence, Fourier series and polynomial approximation. Theoretical development of differentiation and Riemann integration. Required for all mathematics majors. Additional work required for graduate students. (May not be taken for graduate credit by graduate students in the Department of Mathematics.) Offered as MATH 322 and MATH 422. Prereq: MATH 321.

MATH 324. Introduction to Complex Analysis. 3 Units.
Properties, singularities, and representations of analytic functions, complex integration. Cauchy's theorems, series residues, conformal mapping and analytic continuation. Riemann surfaces. Relevance to the theory of physical problems. Prereq: MATH 224 or MATH 228.

MATH 327. Convexity and Optimization. 3 Units.
Introduction to the theory of convex sets and functions and to the extremes in problems in areas of mathematics where convexity plays a role. Among the topics discussed are basic properties of convex sets (extreme points, facial structure of polytopes), separation theorems, duality and polars, properties of convex functions, minima and maxima of convex functions over convex set, various optimization problems. Offered as MATH 327, MATH 427, and OPRE 427. Prereq: MATH 223 or MATH 227.

MATH 330. Introduction to Scientific Computing. 3 Units.
An introductory survey to Scientific Computing from principles to applications. Topics which will be covered in the course include: solution of linear systems and least squares, approximation and interpolation, solution of nonlinear systems, numerical integration and differentiation, and numerical solution of differential equations. Projects where the numerical methods are used to solve problems from various application areas will be assigned throughout the semester. Prereq: MATH 224 or MATH 228. Coreq: MATH 201 or MATH 307.

MATH 332. Equations that Changed the World. 3 Units.
This course will introduce students to some of the fundamental equations that changed the worlds. One equation a week, the students will investigate the mathematics behind some of the most influential equations or ideas, e.g., the Fourier Transform, Maxwell's equations, Schrödinger's equation and the wave equation. Students will research the scientific and social climate in which the equations emerged, and report the impact that the equations have had on the way we see the world and live our lives today. The class will alternate between lectures, where the instructor introduce the mathematical background needed to state and understand for the equation, and presentations, in which the students will present the results of their investigations. The students will be required to write a term paper related to a particular equation and to give a final presentation. The grading will address both the mathematical maturity of the students and the organization and presentation of the paper. Counts as a SAGES Departmental Seminar course. Prereq: (MATH 223 or MATH 227) and (MATH 224 or MATH 228).

MATH 333. Mathematics and Brain. 3 Units.
This course is intended for upper level undergraduate students in Mathematics, Cognitive Science, Biomedical Engineering, Biology or Neuroscience who have an interest in quantitative investigation of the brain and its functions. Students will be introduced to a variety of mathematical techniques needed to model and simulate different brain functions, and to analyze the results of the simulations and of available measured data. The mathematical exposition will be followed--when appropriate--by the corresponding implementation in Matlab. The course will cover some basic topics in the mathematical aspects of differential equations, electromagnetism, Inverse problems and imaging related to brain functions. Validation and falsification of the mathematical models in the light of available experimental data will be addressed. This course will be a first step towards organizing the different brain investigative modalities within a unified mathematical framework. In addition to traditional classroom and assignment work, the course includes a significant writing and discussion component. Counts as a Disciplinary Communication course. Counts as a SAGES Departmental Seminar course. Prereq: MATH 224 or MATH 228.

MATH 338. Introduction to Dynamical Systems. 3 Units.
Nonlinear discrete dynamical systems in one and two dimensions. Chaotic dynamics, elementary bifurcation theory, hyperbolicity, symbolic dynamics, structural stability, stable manifold theory. Prereq: MATH 223 or MATH 227.

MATH 343. Theoretical Computer Science. 3 Units.
Introduction to different classes of automata and their correspondence to different classes of formal languages and grammars, computability, complexity and various proof techniques. Offered as CSDS 343 and MATH 343. Prereq: MATH 304 and CSDS 310.

MATH 345. Introduction to Linear Partial Differential Equations. 3 Units.
Introduction to basic concepts of the linear partial differential equations arising in physics and engineering applications. Heat equations, Wave equations, Laplace's equation. Sturm-Liouville problems. Separation of variables, Boundary value problems. Fourier Series, Bessel functions. Orthogonal function expansions. Green's function methods, transform methods. Prereq: MATH 224 or MATH 228.

MATH 351. Senior Project for the Mathematics and Physics Program. 2 Units.
A two-semester course (2 credits per semester) in the joint B.S. in Mathematics and Physics program. Project based on numerical and/or theoretical research under the supervision of a mathematics faculty member, possibly jointly with a faculty member from physics. Study of the techniques utilized in a specific research area and of recent literature associated with the project. Work leading to meaningful results which are to be presented as a term paper and an oral report at the end of the second semester. Supervising faculty will review progress with the student on a regular basis, including detailed progress reports made twice each semester, to ensure successful completion of the work. Counts as a SAGES Senior Capstone course.

MATH 352. Mathematics Capstone. 3 Units.
Students pursue theoretical, experimental, or teaching research under the supervision of a Capstone Advisor--ordinarily a member of the MAMS Department faculty. Results and conclusions of the project are summarized in written form and in a public presentation, e.g., in the annual MAMS Capstone Symposium, or in the CWRU Intersections Symposium and Poster Sessions. In order to register, a student must first obtain the consent of a Capstone Advisor. Students are strongly encouraged to begin well in advance of registration to initiate discussions with a potential Capstone Advisor. Before granting approval, an advisor may require a Capstone Proposal outlining the goals, expected background, methodology and time frame of the project. The determination as to whether the expectations for the Capstone Project and for the UGER Capstone Requirement have been met are the sole responsibility of the Capstone Advisor. Students who look for a more comprehensive research experience are encouraged to contact the potential capstone advisor one semester before, and, in agreement with the advisor, take a MATH 301 independent study course (1-3 cr) to properly prepare for the project. Counts as a Capstone Project course. Counts as a SAGES Senior Capstone course.

MATH 356. Math in Machine Learning. 3 Units.
The goal of this topic course is to provide students with essential mathematical background for future practice and study in the field of data sciences and machine learning, including modern deep learning architecture. The course is organized as along the following three major themes: (1) Dimensionality Reduction and Transforms, which focus on Singular Value Decomposition (SVD) and Fourier Transform; (2) Machine Learning and Data Analysis, which includes Regression and Model Selection, Clustering and Classification, and Neural Networks; and (3) Dynamic Mode Decomposition. These themes represents important mathematical tools for machine learning and data analysis. In this course, students are also expected to review fundamental concepts in mathematics for machine learning and data sciences, including linear algebra, vector differential calculus, basic probability and optimization. Additionally, students will learn basic Python scripting and how to set up machine learning and deep learning environments for practical applications. Prereq: (MATH 224 or MATH 228) and (MATH 201 or MATH 307).

MATH 357. Mathematical Modeling Across the Sciences. 3 Units.
A three credit course on mathematical modeling as it applies to the origins sciences. Students gain practical experience in a wide range of techniques for modeling research questions in cosmology and astrophysics, integrative evolutionary biology (including physical anthropology, ecology, paleontology, and evolutionary cognitive science), and planetary science and astrobiology. Offered as ORIG 301, ORIG 401 and MATH 357. Prereq: ORIG 201, ORIG 202, BIOL 225, MATH 122, CHEM 106 and (PHYS 122 or PHYS 124).

MATH 358. Mathematical Modeling. 3 Units.
The course is organized as along the following three major themes: (1)Optimization (2)Dynamical modeling (3)Probabilistic methods. These themes represents important mathematical tools for modeling. Starting with practical problems we shall develop mathematical models within each category and analyze them using rigorous and numerical methods. Sensitivity analysis (dependence of the solution on given data) will be an important part of the interpretation of the solution. Counts as a Disciplinary Communication course. Prereq: MATH 224 or MATH 228.

MATH 361. Introduction to Topology. 3 Units.
Metric spaces, topological spaces, and continuous functions. Products; quotients. Compactness, connectedness, path connectedness. Topological manifolds. Fundamental groups; covering spaces. Offered as MATH 361 and MATH 461. Prereq: MATH 307 or MATH 308 or MATH 321.

MATH 363. Knot Theory. 3 Units.
An introduction to the mathematical theory of knots and links, with emphasis on the modern combinatorial methods. Reidemeister moves on link projections, ambient and regular isotopies, linking number tricolorability, rational tangles, braids, torus knots, seifert surfaces and genus, the knot polynomials (bracket, X, Jones, Alexander, HOMFLY), crossing numbers of alternating knots and amphicheirality. Connections to theoretical physics, molecular biology, and other scientific applications will be pursued in term projects, as appropriate to the background and interests of the students. Prereq: MATH 223 or MATH 227.

MATH 364. Geometry I. 3 Units.
An introduction to the various two-dimensional geometries, including Euclidean, spherical, hyperbolic, projective, and affine. The course will examine the axiomatic basis of geometry, with an emphasis on transformations. Topics include the parallel postulate and its alternatives, isometrics and transformation groups, tilings, the hyperbolic plane and its models, spherical geometry, affine and projective transformations, and other topics. We will examine the role of complex and hypercomplex numbers in the algebraic representation of transformations. The course is self-contained. Counts as a SAGES Departmental Seminar course. Prereq: MATH 224.

MATH 365. Introduction To Algebraic Geometry. 3 Units.
This is a first introduction to algebraic geometry--the study of solutions of polynomial equations-- for advanced undergraduate students. Recent applications of this large and important area include number theory, combinatorics, theoretical physics, robotics, cryptology and coding theory. The contents of the course may vary from one semester to another, and may include, for example: the classical theory of algebraic curves in the setting of affine and projective planes over the real or complex fields; affine and projective equivalence; invariants; tangents; singularities; intersection multiplicities; resultants and Bezout's Theorem; linear systems; rational curves; flexes and group structure on a cubic. Prereq: MATH 307 and Coreq: MATH 308.

MATH 376. Mathematical Analysis of Biological Models. 3 Units.
This course focuses on the mathematical methods used to analyze biological models, with examples drawn largely from ecology but also from epidemiology, developmental biology, and other areas. Mathematical topics include equilibrium and stability in discrete and continuous time, some aspects of transient dynamics, and reaction-diffusion equations (steady state, diffusive instabilities, and traveling waves). Biological topics include several "classic" models, such as the Lotka-Volterra model, the Ricker model, and Michaelis-Menten/type II/saturating responses. The emphasis is on approximations that lead to analytic solutions, not numerical analysis. An important aspect of this course is translating between verbal and mathematical descriptions: the goal is not just to solve mathematical problems but to extract biological meaning from the answers we find. Offered as BIOL 306 and MATH 376. Prereq: BIOL 300 or MATH 224 or consent of instructor.

MATH 378. Computational Neuroscience. 3 Units.
Computer simulations and mathematical analysis of neurons and neural circuits, and the computational properties of nervous systems. Students are taught a range of models for neurons and neural circuits, and are asked to implement and explore the computational and dynamic properties of these models. The course introduces students to dynamical systems theory for the analysis of neurons and neural learning, models of brain systems, and their relationship to artificial and neural networks. Term project required. Students enrolled in MATH 478 will make arrangements with the instructor to attend additional lectures and complete additional assignments addressing mathematical topics related to the course. Recommended preparation: MATH 223 and MATH 224 or BIOL 300 and BIOL 306. Offered as BIOL 378, COGS 378, MATH 378, BIOL 478, CSDS 478, EBME 478, ECSE 478, MATH 478 and NEUR 478.

MATH 380. Introduction to Probability. 3 Units.
Combinatorial analysis. Permutations and combinations. Axioms of probability. Sample space and events. Equally likely outcomes. Conditional probability. Bayes' formula. Independent events and trials. Discrete random variables, probability mass functions. Expected value, variance. Bernoulli, binomial, Poisson, geometric, negative binomial random variables. Continuous random variables, density functions. Expected value and variance. Uniform, normal, exponential, Gamma random variables. The De Moivre-Laplace limit theorem. Joint probability mass functions and densities. Independent random variables and the distribution of their sums. Covariance. Conditional expectations and distributions (discrete case). Moment generating functions. Law of large numbers. Central limit theorem. Additional topics (time permitting): the Poisson process, finite state space Markov chains, entropy. Prereq: MATH 223 or MATH 227.

MATH 382. High Dimensional Probability. 3 Units.
Behavior of random vectors, random matrices, and random projections in high dimensional spaces, with a view toward applications to data sciences. Topics include tail inequalities for sums of independent random variables, norms of random matrices, concentration of measure, and bounds for random processes. Applications may include structure of random graphs, community detection, covariance estimation and clustering, randomized dimension reduction, empirical processes, statistical learning, and sparse recovery problems. Additional work is required for graduate students. Offered as MATH 382, MATH 482, STAT 382 and STAT 482. Prereq: MATH 307 and (MATH 380 or STAT 345 or STAT 445).

MATH 386. Quantum Computing, Information, and Devices. 3 Units.
An introduction to the math, physics, engineering, and computer science underlying the rapidly emerging fields of quantum computing, quantum information, and quantum devices. The course is taught by a group of faculty from physics, engineering, computer science, and math, and is geared towards students with diverse backgrounds and interests in these fields. Students will select a concentration in one of these four areas, and the coursework, while still covering all topics, will be adjusted to focus on the selected area in the most detail. Note that the listed prerequisites depend on choice of concentration. Topics will include: 1. (Mathematics) Introduction to linear algebra, convex geometry, fundamental theory of quantum information. 2. (Physics) Introduction to the quantum mechanics of two-level systems (qubits). Survey of physics and materials for qubit technologies. 3. (Computer Science) Basic quantum gates and circuits, introduction to the theory of algorithms, survey of quantum algorithms. 4. (Engineering) Quantum architectures, mapping algorithms onto circuits. The course consists of lectures, homework, and group projects. Group projects will aim to synthesize the diverse backgrounds of the students and instructors to capture the interdisciplinary nature of the field. Students taking the course for graduate credit will complete an additional literature research project and presentation, in addition to enhanced problem sets. Offered as CSDS 386, CSDS 486, ECSE 386, ECSE 486, MATH 386, MATH 486, PHYS 386, and PHYS 486. Prereq: (MATH 223 or MATH 227) and (MATH 224 or MATH 228) and (MATH 201 or MATH 307) and (PHYS 122 or PHYS 124) and PHYS 221.

MATH 394. Introduction to Information Theory. 3 Units.
This course is intended as an introduction to information and coding theory with emphasis on the mathematical aspects. It is suitable for advanced undergraduate and graduate students in mathematics, applied mathematics, statistics, physics, computer science and electrical engineering. Course content: Information measures-entropy, relative entropy, mutual information, and their properties. Typical sets and sequences, asymptotic equipartition property, data compression. Channel coding and capacity: channel coding theorem. Differential entropy, Gaussian channel, Shannon-Nyquist theorem. Information theory inequalities (400 level). Additional topics, which may include compressed sensing and elements of quantum information theory. Recommended preparation: MATH 201 or MATH 307. Offered as MATH 394, CSDS 394, ECSE 394, MATH 494, CSDS 494 and ECSE 494. Prereq: MATH 223 and MATH 380 or requisites not met permission.

MATH 400. Mathematics/Statistics Teaching Practicum. 1 Unit.
Practicum for teaching college mathematics and statistics. Includes preparation of syllabi, exams, lectures. Grading, alternative teaching styles, use of technology, interpersonal relations and motivation. Handling common problems and conflicts. Offered as MATH 400 and STAT 400.

MATH 401. Abstract Algebra I. 3 Units.
Basic properties of groups, rings, modules and fields. Isomorphism theorems for groups; Sylow theorem; nilpotency and solvability of groups; Jordan-Holder theorem; Gauss lemma and Eisenstein's criterion; finitely generated modules over principal ideal domains with applications to abelian groups and canonical forms for matrices; categories and functors; tensor product of modules, bilinear and quadratic forms; field extensions; fundamental theorem of Galois theory, solving equations by radicals. Prereq: MATH 308.

MATH 402. Abstract Algebra II. 3 Units.
A continuation of MATH 401. Prereq: MATH 401.

MATH 405. Advanced Matrix Analysis. 3 Units.
An advanced course in linear algebra and matrix theory. Topics include variational characterizations of eigenvalues of Hermitian matrices, matrix and vector norms, characterizations of positive definite matrices, singular value decomposition and applications, perturbation of eigenvalues. This course is more theoretical than MATH 431, which emphasizes computational aspects of linear algebra Prereq: MATH 307.

MATH 406. Mathematical Logic and Model Theory. 3 Units.
Propositional calculus and quantification theory; consistency and completeness theorems; Gödel incompleteness results and their philosophical significance; introduction to basic concepts of model theory; problems of formulation of arguments in philosophy and the sciences. Offered as PHIL 306, MATH 406 and PHIL 406.

MATH 408. Introduction to Cryptology. 3 Units.
Introduction to the mathematical theory of secure communication. Topics include: classical cryptographic systems; one-way and trapdoor functions; RSA, DSA, and other public key systems; Primality and Factorization algorithms; birthday problem and other attack methods; elliptic curve cryptosystems; introduction to complexity theory; other topics as time permits. Recommended preparation: MATH 303.

MATH 413. Graph Theory. 3 Units.
Building blocks of a graph, trees, connectivity, matchings, coverings, planarity, NP-complete problems, random graphs, and expander graphs; various applications and algorithms. Prereq: MATH 201 or MATH 307.

MATH 418. Category Theory. 3 Units.
Category theory is the rigorous study of the consequences of taking the notation of an arrow for a function seriously. Instead of emphasizing that a function has variables to evaluate via a formula, category theory posits that the most important features of how we use functions are governed by the axioms for composition and the consequences of those axioms. Category theory has roots in algebraic topology and homological algebra, but also has deep connections with topics as diverse as algebraic geometry, logic, and metric spaces. The goal of this course is to give students an introduction to category theory an an independent subject of study. As such, this course will view category theory as a kind of algebra, and so familiarity with other algebraic structures (groups, rings, vector spaces), good proof-writing technique, and comfort with a high degree of abstraction are the primary prerequisites. Examples from a wide variety of other areas of mathematics will be given throughout, but no subject-specific knowledge from these areas will be required. Prereq: Graduate Student standing or MATH 308 or MATH 309.

MATH 419. Applied Probability and Stochastic Processes for Biology. 3 Units.
Applications of probability and stochastic processes to biological systems. Mathematical topics will include: introduction to discrete and continuous probability spaces (including numerical generation of pseudo random samples from specified probability distributions), Markov processes in discrete and continuous time with discrete and continuous sample spaces, point processes including homogeneous and inhomogeneous Poisson processes and Markov chains on graphs, and diffusion processes including Brownian motion and the Ornstein-Uhlenbeck process. Biological topics will be determined by the interests of the students and the instructor. Likely topics include: stochastic ion channels, molecular motors and stochastic ratchets, actin and tubulin polymerization, random walk models for neural spike trains, bacterial chemotaxis, signaling and genetic regulatory networks, and stochastic predator-prey dynamics. The emphasis will be on practical simulation and analysis of stochastic phenomena in biological systems. Numerical methods will be developed using a combination of MATLAB, the R statistical package, MCell, and/or URDME, at the discretion of the instructor. Student projects will comprise a major part of the course. Offered as BIOL 319, ECSE 319, MATH 319, SYBB 319, BIOL 419, EBME 419, MATH 419, PHOL 419, and SYBB 419.

MATH 421. Fundamentals of Analysis I. 3 Units.
Abstract mathematical reasoning in the context of analysis in Euclidean space. Introduction to formal reasoning, sets and functions, and the number systems. Sequences and series; Cauchy sequences and convergence. Required for all mathematics majors. Additional work required for graduate students. (May not be taken for graduate credit by graduate students in the Department of Mathematics.) Offered as MATH 321 and MATH 421.

MATH 422. Fundamentals of Analysis II. 3 Units.
Continuation of MATH 321. Point-set topology in metric spaces with attention to n-dimensional space; completeness, compactness, connectedness, and continuity of functions. Topics in sequences, series of functions, uniform convergence, Fourier series and polynomial approximation. Theoretical development of differentiation and Riemann integration. Required for all mathematics majors. Additional work required for graduate students. (May not be taken for graduate credit by graduate students in the Department of Mathematics.) Offered as MATH 322 and MATH 422. Prereq: MATH 321 or MATH 421.

MATH 423. Introduction to Real Analysis I. 3 Units.
General theory of measure and integration. Measures and outer measures. Lebesgue measure on n-space. Integration. Convergence theorems. Product measures and Fubini's theorem. Signed measures. Hahn-Jordan decomposition, Radon-Nikodym theorem, and Lebesgue decomposition. SpaceP-integrable function. Lebesgue differentiation theorem in n-space. Prereq: MATH 322 or MATH 422.

MATH 424. Introduction to Real Analysis II. 3 Units.
Measures on locally compact spaces. Riesz representation theorem. Elements of functional analysis. Normed linear spaces. Hahn-Banach, Banach-Steinhaus, open mapping, closed graph theorems. Weak topologies. Banach-Alaoglu theorem. Function spaces. Stone-Weierstrass and Ascoli theorems. Basic Hilbert space theory. Application to Fourier series. Additional topics: Haar measure on locally compact groups. Prereq: MATH 423.

MATH 425. Complex Analysis I. 3 Units.
Analytic functions. Integration over paths in the complex plane. Index of a point with respect to a closed path; Cauchy's theorem and Cauchy's integral formula; power series representation; open mapping theorem; singularities; Laurent expansion; residue calculus; harmonic functions; Poisson's formula; Riemann mapping theorem. More theoretical and at a higher level than MATH 324. Prereq: MATH 322 or MATH 422.

MATH 427. Convexity and Optimization. 3 Units.
Introduction to the theory of convex sets and functions and to the extremes in problems in areas of mathematics where convexity plays a role. Among the topics discussed are basic properties of convex sets (extreme points, facial structure of polytopes), separation theorems, duality and polars, properties of convex functions, minima and maxima of convex functions over convex set, various optimization problems. Offered as MATH 327, MATH 427, and OPRE 427.

MATH 428. Fourier Analysis and Applications. 3 Units.
Introduction to the mathematical aspects of Fourier analysis and synthesis. Accessible to upper level undergraduates and graduate students in the sciences and engineering. Periodic functions. Fourier series. Convergence theorems. The classical sine and cosine series. General orthogonal systems. Multiple Fourier series. Applications. Fourier integrals and Fourier Transforms. L1 and L2 theory. Inversion theorems. Classical sine and cosine transforms. Multiple Fourier Transform. Spherical symmetry. Other important transforms and representation systems including wavelets. Applications. We will also use some elements of measure theory, those will be reviewed/introduced in the beginning of the semester. Recommended Preparation: MATH 307 and MATH 321. Prereq: Graduate student standing or MATH 224.

MATH 431. Introduction to Numerical Analysis I. 3 Units.
Numerical linear algebra for scientists and engineers. Matrix and vector norms, computer arithmetic, conditioning and stability, orthogonality. Least squares problems: QR factorization, normal equations and Singular Value Decomposition. Direct solution of linear system: Gaussian elimination and Cholesky factorization. Eigenvalues and eigenvectors: the QR algorithm, Rayleigh quotient, inverse iteration. Introduction to iterative methods. Students will be introduced to MATLAB. Prereq: MATH 201 or MATH 307.

MATH 432. Numerical Differential Equations. 3 Units.
Numerical solution of differential equations for scientists and engineers. Solution of ordinary differential equations by multistep and single step methods. Stability, consistency, and convergence. Stiff equations. Finite difference schemes. Introduction to the finite element method. Introduction to multigrid techniques. The diffusion equation: numerical schemes and stability analysis. Introduction to hyperbolic equations. MATLAB will be used in this course. Prereq: MATH 224 or MATH 228.

MATH 433. Numerical Solutions of Nonlinear Systems and Optimization. 3 Units.
The course provides an introduction to numerical solution methods for systems of nonlinear equations and optimization problems. The course is suitable for upper-undergraduate and graduate students with some background in calculus and linear algebra. Knowledge of numerical linear algebra is helpful. Among the topics which will be covered in the course are Nonlinear systems in one variables; Newton's method for nonlinear equations and unconstrained minimization; Quasi-Newton methods; Global convergence of Newton's methods and line searches; Trust region approach; Secant methods; Nonlinear least squares. Prereq: MATH 223 or MATH 227, and MATH 431 or permission.

MATH 435. Ordinary Differential Equations. 3 Units.
A second course in ordinary differential equations. Existence, uniqueness, and continuation of solutions of ODE. Linear systems, fundamental matrix, qualitative methods (phase plane). Dependence on initial data and parameters (Gronwall's inequality, nonlinear variation of parameters). Stability for linear and nonlinear equations, linearization, Poincare-Bendixson theory. Additional topics may include regular and singular perturbation methods, autonomous oscillations, entrainment of forced oscillators, and bifurcations. Prereq: MATH 224 and either MATH 201 or MATH 307.

MATH 439. Bayesian Scientific Computing. 3 Units.
This course will embed numerical methods into a Bayesian framework. The statistical framework will make it possible to integrate a prori information about the unknowns and the error in the data directly into the most efficient numerical methods. A lot of emphasis will be put on understanding the role of the priors, their encoding into fast numerical solvers, and how to translate qualitative or sample-based information--or lack thereof--into a numerical scheme. Confidence on computed results will also be discussed from a Bayesian perspective, at the light of the given data and a priori information. The course should be of interest to anyone working on signal and image processing statistics, numerical analysis and modeling. Recommended Preparation: MATH 431. Offered as MATH 439 and STAT 439.

MATH 440. Computational Inverse Problems. 3 Units.
This course will introduce various computational methods for solving inverse problems under different conditions. First the classical regularization methods will be introduced, and the computational challenges which they pose, will be addressed. Following this, the statistical methods for solving inverse problems will be studied and their computer implementation discussed. We will combine the two approaches to best exploit their potentials. Applications arising from various areas of science, engineering, and medicine will be discussed throughout the course.

MATH 441. Mathematical Modeling. 3 Units.
Mathematics is a powerful language for describing real world phenomena and providing predictions that otherwise are hard or impossible to obtain. The course gives the students pre-requisites for translating qualitative descriptions given in the professional non-mathematical language into the quantitative language for mathematics. While the variety in the subject matter is wide, some general principles and methodologies that a modeler can pursue are similar in many applications. The course focuses on these similarities. The course is based on representative case studies that are discussed and analyzed in the classroom, the emphasis being on general principles of developing and analyzing mathematical models. The examples will be taken from different fields of science and engineering, including life sciences, environmental sciences, biomedical engineering and physical sciences. Modeling relies increasingly on computation, so the students should have basic skills for using computers and programs like Matlab or Mathematica. Prereq: MATH 224 or MATH 228.

MATH 444. Mathematics of Data Mining and Pattern Recognition. 3 Units.
This course will give an introduction to a class of mathematical and computational methods for the solution of data mining and pattern recognition problems. By understanding the mathematical concepts behind algorithms designed for mining data and identifying patterns, students will be able to modify to make them suitable for specific applications. Particular emphasis will be given to matrix factorization techniques. The course requirements will include the implementations of the methods in MATLAB and their application to practical problems. Prereq: MATH 201 or MATH 307.

MATH 445. Introduction to Partial Differential Equations. 3 Units.
Method of characteristics for linear and quasilinear equations. Second order equations of elliptic, parabolic, type; initial and boundary value problems. Method of separation of variables, eigenfunction expansions, Sturm-Liouville theory. Fourier, Laplace, Hankel transforms; Bessel functions, Legendre polynomials. Green's functions. Examples include: heat diffusion, Laplace's equation, wave equations, one dimensional gas dynamics and others. Appropriate for seniors and graduate students in science, engineering, and mathematics. Prereq: MATH 201 or MATH 307 and MATH 224 or MATH 228.

MATH 446. Numerical Methods for Partial Differential Equations. 3 Units.
This course is an introduction to numerical methods of PDEs, and in particular, to finite element methods (FEM), emphasizing the interconnection between the functional analytic viewpoint of PDEs and the practical and effective computation of the numerical approximations. In particular, the emphasis is on showing that many of the useful and elegant ideas in finite dimensional linear algebra have a natural counterpart in the infinite dimensional setting of Hilbert spaces, and that the same techniques that guarantee the existence and uniqueness of the solutions in fact provide also stable computational methods to approximate the solutions. The topics covered in this course include Fourier analysis, weak derivatives, weak forms, generalized functions; Sobolev spaces, trace theorem, compact embedding theorems, Poincare inequalities; Riesz theory, Fredholm theory; Finite Element Method (FEM): Grid generation, existence, stability and convergence of solutions for elliptic problems; Semi-discretization of parabolic and hyperbolic equations; Stiffness; Numerical solution of linear systems by iterative methods. A quintessential part of this course comprises numerical implementation of the finite element method. Matlab is used as the programming tool both in demonstrations and examples in the class as well as in home assignments. Recommended Preparation: linear algebra, multivariate calculus, and ordinary differential equations.

MATH 447. Computational Fluid Mechanics. 3 Units.
This course covers the fundamentals of numerical algorithms for modeling dynamics of fluid flow computationally. Includes various approaches to discretize time and space on structured and unstructured grids with a variety of boundary conditions. Involves programming of basic CFD codes in MATLAB to test example problems in fluid mechanics with different discretization schemes. Topics include finite difference, finite element, and spectral techniques for numerical solutions of partial differential equations. Prereq: Graduate student standing.

MATH 449. Dynamical Models for Biology and Medicine. 3 Units.
Introduction to discrete and continuous dynamical models with applications to biology and medicine. Topics include: population dynamics and ecology; models of infectious diseases; population genetics and evolution; biological motion (reaction-diffusion and chemotaxis); Molecular and cellular biology (biochemical kinetics, metabolic pathways, immunology). The course will introduce students to the basic mathematical concepts and techniques of dynamical systems theory (equilibria, stability, bifurcations, discrete and continuous dynamics, diffusion and wave propagation, elements of system theory and control). Mathematical exposition is supplemented with introduction to computer tools and techniques (Mathematica, Matlab). Prereq: MATH 224 or MATH 228 or BIOL 300 or EBME 300, and MATH 201.

MATH 461. Introduction to Topology. 3 Units.
Metric spaces, topological spaces, and continuous functions. Products; quotients. Compactness, connectedness, path connectedness. Topological manifolds. Fundamental groups; covering spaces. Offered as MATH 361 and MATH 461. Prereq: MATH 307 or MATH 308 or MATH 321 or Graduate Student standing.

MATH 462. Algebraic Topology. 3 Units.
The fundamental group and covering spaces; van Kampen's theorem. Higher homotopy groups; long-exact sequence of a pair. Homology theory; chain complexes; short and long exact sequences; Mayer-Vietoris sequence. Homology of surfaces and complexes; applications. Prereq: MATH 461.

MATH 465. Differential Geometry. 3 Units.
Manifolds and differential geometry. Vector fields; Riemannian metrics; curvature; intrinsic and extrinsic geometry of surfaces and curves; structural equations of Riemannian geometry; the Gauss-Bonnet theorem. Prereq: MATH 321.

MATH 467. Differentiable Manifolds. 3 Units.
Differentiable manifolds and structures on manifolds. Tangent and cotangent bundle; vector fields; differential forms; tensor calculus; integration and Stokes' theorem. May include Hamiltonian systems and their formulation on manifolds; symplectic structures; connections and curvature; foliations and integrability. Prereq: MATH 322.

MATH 473. Introduction to Mathematical Image Processing and Computer Vision. 3 Units.
This course introduces fundamental mathematics techniques for image processing and computer vision (IPCV). It is accessible to upper level undergraduate and graduate students from mathematics, sciences, engineering and medicine. Topics include but are not limited to image denoising, contrast enhancement, image compression, image segmentation and pattern recognition. Main tools are discrete Fourier analysis and wavelets, plus some statistics, optimization and a little calculus of variation and partial differential equations if time permitting. Students gain a solid theoretical background in IPCV modeling and computing, and master hands-on application experiences. Upon completion of the course, students will have clear understanding of classical methods, which will help them develop new methodical approaches for imaging problems arising in a variety of fields. Recommended preparation: Some coursework in scientific computing and ability to program in (or willingness to learn) a language such as Matlab or C/C++. Prereq: MATH 330 or MATH 431 or equivalent.

MATH 478. Computational Neuroscience. 3 Units.
Computer simulations and mathematical analysis of neurons and neural circuits, and the computational properties of nervous systems. Students are taught a range of models for neurons and neural circuits, and are asked to implement and explore the computational and dynamic properties of these models. The course introduces students to dynamical systems theory for the analysis of neurons and neural learning, models of brain systems, and their relationship to artificial and neural networks. Term project required. Students enrolled in MATH 478 will make arrangements with the instructor to attend additional lectures and complete additional assignments addressing mathematical topics related to the course. Recommended preparation: MATH 223 and MATH 224 or BIOL 300 and BIOL 306. Offered as BIOL 378, COGS 378, MATH 378, BIOL 478, CSDS 478, EBME 478, ECSE 478, MATH 478 and NEUR 478.

MATH 482. High Dimensional Probability. 3 Units.
Behavior of random vectors, random matrices, and random projections in high dimensional spaces, with a view toward applications to data sciences. Topics include tail inequalities for sums of independent random variables, norms of random matrices, concentration of measure, and bounds for random processes. Applications may include structure of random graphs, community detection, covariance estimation and clustering, randomized dimension reduction, empirical processes, statistical learning, and sparse recovery problems. Additional work is required for graduate students. Offered as MATH 382, MATH 482, STAT 382 and STAT 482. Prereq: Graduate student standing.

MATH 486. Quantum Computing, Information, and Devices. 3 Units.
An introduction to the math, physics, engineering, and computer science underlying the rapidly emerging fields of quantum computing, quantum information, and quantum devices. The course is taught by a group of faculty from physics, engineering, computer science, and math, and is geared towards students with diverse backgrounds and interests in these fields. Students will select a concentration in one of these four areas, and the coursework, while still covering all topics, will be adjusted to focus on the selected area in the most detail. Note that the listed prerequisites depend on choice of concentration. Topics will include: 1. (Mathematics) Introduction to linear algebra, convex geometry, fundamental theory of quantum information. 2. (Physics) Introduction to the quantum mechanics of two-level systems (qubits). Survey of physics and materials for qubit technologies. 3. (Computer Science) Basic quantum gates and circuits, introduction to the theory of algorithms, survey of quantum algorithms. 4. (Engineering) Quantum architectures, mapping algorithms onto circuits. The course consists of lectures, homework, and group projects. Group projects will aim to synthesize the diverse backgrounds of the students and instructors to capture the interdisciplinary nature of the field. Students taking the course for graduate credit will complete an additional literature research project and presentation, in addition to enhanced problem sets. Offered as CSDS 386, CSDS 486, ECSE 386, ECSE 486, MATH 386, MATH 486, PHYS 386, and PHYS 486. Prereq: (MATH 223 or MATH 227) and (MATH 224 or MATH 228) and (MATH 201 or MATH 307) and (PHYS 122 or PHYS 124) and PHYS 221.

MATH 494. Introduction to Information Theory. 3 Units.
This course is intended as an introduction to information and coding theory with emphasis on the mathematical aspects. It is suitable for advanced undergraduate and graduate students in mathematics, applied mathematics, statistics, physics, computer science and electrical engineering. Course content: Information measures-entropy, relative entropy, mutual information, and their properties. Typical sets and sequences, asymptotic equipartition property, data compression. Channel coding and capacity: channel coding theorem. Differential entropy, Gaussian channel, Shannon-Nyquist theorem. Information theory inequalities (400 level). Additional topics, which may include compressed sensing and elements of quantum information theory. Recommended preparation: MATH 201 or MATH 307. Offered as MATH 394, CSDS 394, ECSE 394, MATH 494, CSDS 494 and ECSE 494.

MATH 497. Stochastic Models: Time Series and Markov Chains. 3 Units.
Introduction to stochastic modeling of data. Emphasis on models and statistical analysis of data with a significant temporal and/or spatial structure. This course will analyze time and space dependent random phenomena from two perspectives: Stationary Time Series: Spectral representation of deterministic signals, autocorrelation. Power spectra. Transmission of stationary signals through linear filters. Optimal filter design, signal-to-noise ratio. Gaussian signals and correlation matrices. Spectral representation and computer simulation of stationary signals. Discrete Markov Chains: Transition matrices, recurrences and the first step analysis. Steady rate. Recurrence and ergodicity, empirical averages. Long run behavior, convergence to steady state. Time to absorption. Eigenvalues and nonhomogeneous Markov chains. Introduction to Gibbs fields and Markov Chain Monte Carlo (MCMC). This course is related to STAT 538 but can be taken independently of it. Offered as: MATH 497 and STAT 437. Prereq: STAT 312 or STAT 312R or STAT 313 or STAT 332 or STAT 333 or STAT 345 or MATH 380 or MATH 491 or (STAT 243 and STAT 244) or Requisites Not Met permission.

MATH 499. Special Topics. 3 Units.
Special topics in mathematics.

MATH 528. Analysis Seminar. 1 - 3 Units.
Continuing seminar on areas of current interest in analysis. Allows graduate and advanced undergraduate students to become involved in research. Topics will reflect interests and expertise of the faculty and may include functional analysis, convexity theory, and their applications. May be taken more than once for credit. Consent of department required.

MATH 598. Stochastic Models: Diffusive Phenomena and Stochastic Differential Equations. 3 Units.
Introduction to stochastic modeling of data. Emphasis on models and statistical analysis of data with significant temporal and/or spatial structure. This course will analyze time and space dependent random phenomena from two perspectives: Brownian motion and diffusive processes: Classification of stochastic processes, finite dimensional distributions, random walks and their scaling limits, Brownian motion and its paths properties, general diffusive processes, Fokker-Planck-Kolmogorov equations, Poisson and point processes, heavy tail diffusions, Levy processes, tempered stable diffusions. Stochastic calculus and stochastic differential equations: Wiener random integrals, mean-square theory, Brownian stochastic integrals and Ito formula, stochastic integrals for Levy processes, martingale property, basic theory and applications of stochastic differential equations. This course is related to STAT 437 but can be taken independently of it. Offered as MATH 598 and STAT 538.

MATH 601. Reading and Research Problems. 1 - 18 Units.
Presentation of individual research, discussion, and investigation of research papers in a specialized field of mathematics.

MATH 651. Thesis (M.S.). 1 - 18 Units.
(Credit as arranged.)

MATH 701. Dissertation (Ph.D.). 1 - 9 Units.
(Credit as arranged.) Prereq: Predoctoral research consent or advanced to Ph.D. candidacy milestone.